UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	:	BGE 2113
COURSE	:	NUMERICAL METHOD
SEMESTER/SESSION	:	1 – 2022/2023
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **FIVE (5)** questions in SECTION A, **THREE (3)** questions in SECTION B and **TWO (2)** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Use all calculations in **FOUR (4)** decimal places.
4. Write legibly and draw sketches wherever required.
5. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

The graph of $f(x)=2x\cos 2x-(x-2)^2$ over the interval is shown in Figure 1 below. Find the least positive root of $f(x)$ below with $|x_1-x_0|=1$ by using Secant method. Iterate until $|f(x_{i+1})|<0.005$.

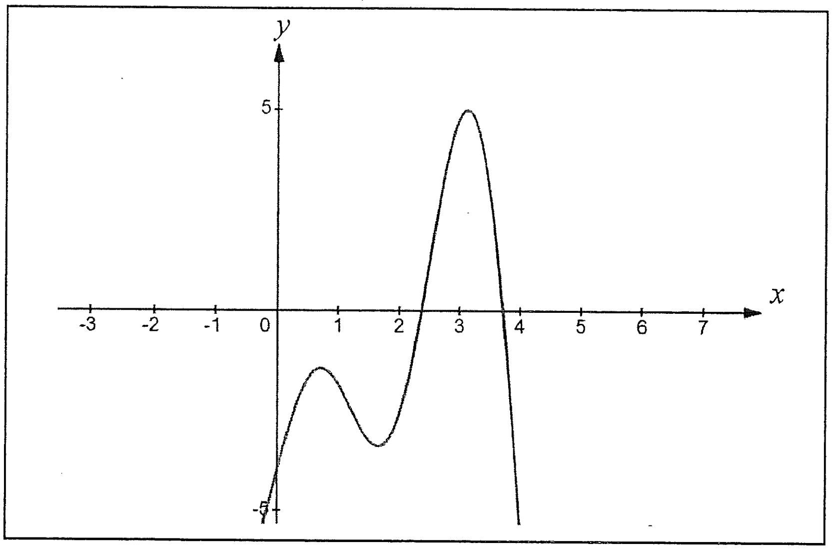

Figure 1: A graph of $f(x)=2x\cos 2x-(x-2)^2$

(13 marks)

QUESTION 2

Given a system of linear equation,

$$10x+3y-2z=17$$
$$x+6z=7$$
$$2x-6y+z=27$$

Use the Gauss Elimination Method to solve the above system of linear equations. (10 marks)

QUESTION 3

Evaluate the following definite integral,

$$\int_1^7 \sqrt{1+x+x^2}\,dx$$

by using the Trapezoidal Rule with 6 subintervals. (6 marks)

QUESTION 4

(a) Approximate the values of the first derivative of the following function,

$$f(x) = e^{2x}\ln\sqrt{x}$$

at point $x = 2.5$ with step size, $h = 0.1$ using:

i. 2-point forward, (2 marks)

ii. 3-point forward, (2 marks)

iii 3-point central, (2 marks)

iv. 5-point central difference formula. (2 marks)

(b) Calculate the percent relative error for each approximation obtained in (a) using the analytical solution, $f'(x) = e^{2x}\left(\ln x + \frac{1}{2x}\right)$. (5 marks)

QUESTION 5

Given a data set of sine function in Table 1.

Table 1

x	0.5	1.5	2.5
$f(x)$	0.4794	0.9975	0.5985

Find the approximation value of $f(2)$ by using second order Lagrange interpolation polynomial method. (8 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Solve the following system of linear equation below by using LU Factorization method.

$$\begin{pmatrix} 10 & -2 & 1 \\ 3 & 4 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \\ -14 \end{pmatrix}$$

(12 marks)

QUESTION 2

Determine the value of a, b, c and d in the table below by using the Newton's divided difference method.

i	x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$
0	1.0	0.45	-0.8475	d	-0.3208
1	1.4	a	0.32	1.625	
2	1.6	b	c		
3	2.0	0.693			

(7 marks)

QUESTION 3

Compute the following definite integral

$$\frac{1}{5}\int_{-2}^{0} x\left(x^3 - 1\right)dx$$

(a) analytically. (2 marks)

(b) using an appropriate Simpson's rule with step size, $h = 0.25$ and give a reason why you choose that method (8 marks)

(c) calculate the percent relative error obtained in (b). (1 mark)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

The following Table 2 gives the values of distance traveled by a car at various times from a tollgate at highway.

Table 2

Time, t (minute)	3	5	7	9	11	13
Distance, x (km)	4.6	6.03	12.966	14.885	18.904	21.041

Estimate the acceleration of the car at time $t = 9$ minutes by using all the appropriate formulas in Table of Difference with a suitable value of step size, h.

(6 marks)

QUESTION 2

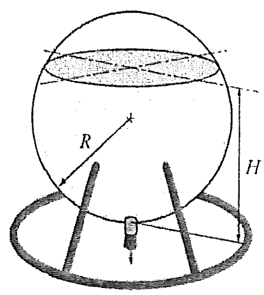

Figure 2: A spherical water tank

Figure 2 shows a spherical water tank of radius, $R = 4\ m$ is emptied through a small circulation hole of radius, $r = 0.2\,m$ at the bottom. The top of tank is open to the atmosphere. The instantaneous water level, H in the tank (measured from the bottom of the tank, at the drain) can be determined from the solution of the following ordinary differential equation (ODE):

$$\frac{dH}{dt} = -\frac{r^2\sqrt{2gH}}{2RH - H^2},$$

where $g = 9.81\ m/\sec^2$ and the initial height is $6.5\ m$. Determine the height of water for $0 \le t \le 150$ by using Heun's method with time step, $50\ \sec$. (14 marks)

-- **END OF QUESTION** --

FORMULA

Percent Relative Error	$\left\lvert \dfrac{\text{Exact} - \text{Approximate}}{\text{Exact}} \right\rvert \times 100\%$
Secant method	$x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$ for $i = 0,1,2,3,\ldots$
LU Factorization method	$A = LU$ $LY = B$ $UX = Y$
Lagrange Interpolation Polynomial method	$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$ for $i = 0,1,2,3,\ldots$ where $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \dfrac{(x - x_j)}{(x_i - x_j)}$
Newton's Divided-Difference method	$f_n(x) = f(x_0) + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots$ $+ (x - x_0)(x - x_1)\ldots(x - x_{n-1}) f[x_0, x_1, \ldots, x_{n-1}, x_n]$ where $f[x_i, x_{i+1}] = \dfrac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$ $f[x_i, x_{i+1}, x_{i+2}] = \dfrac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$ and so on.
Trapezoidal Rule	$\int_a^b f(x)\,dx \approx \dfrac{h}{2}\left[f_0 + f_n + 2\sum_{i=1}^{n-1} f_i \right]$
Simpson's Rule	$\int_a^b f(x)\,dx \approx \dfrac{h}{3}\left[f_0 + f_n + 4\underbrace{\sum_{i=1}^{n-1} f_i}_{\text{Odd term}} + 2\underbrace{\sum_{i=2}^{n-2} f_i}_{\text{Even term}} \right]$ $\int_a^b f(x)\,dx \approx \dfrac{3h}{8}\left[f_0 + f_n + 3\sum_{i=1}^{n-1} f_i + \underbrace{2\sum_{i=3}^{n-3} f_i}_{\text{multiple of three}} \right]$
Runge-Kutta Order 2 (RK2) method	$y_{i+1} = y_i + \dfrac{h}{2}(k_1 + k_2)$ where $k_1 = f(x_i, y_i)$ $k_2 = f(x_i + h, y_i + k_1 h)$

Table of Difference formula

First Derivative	2- Point	Forward	$f'(x) \approx \frac{f(x+h)-f(x)}{h}$
		Backward	$f'(x) \approx \frac{f(x)-f(x-h)}{h}$
	3-Point	Central	$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$
		Forward	$f'(x) \approx \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$
		Backward	$f'(x) \approx \frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$
	5-Point		$f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$
Second Derivative	3-Point	Central	$f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$
	5-Point		$f''(x) \approx \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$